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TWELFTH PROGRESS REPORT

on

CALIBRATION AND EVALUATION OF SKYLAB ALTIMETRY FOR  
GEODETTIC DETERMINATION OF THE GEOID (Contract NAS9-13276,  
EPN 440), February 1 to February 28, 1974

to

NASA Johnson Space Center  
Principal Investigation Management Office  
Houston, Texas 77058

from

BATTELLE  
Columbus Laboratories

March 15, 1974

Prepared by: D. M. J. Fubara (Co-Investigator)

A. G. Mourad (Principal Investigator)  
Z. H. Byrns, Code TF6 - NASA/JSC Technical Monitor

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PROGRESS

During this period, we (1) completed the geodetic processing and analysis of the updated and improved orbital and altimeter data for SL-2 EREP pass 9, as received from NASA/Wallops, (2) reprocessed and analyzed the NASA/JSC version of the same data set, (3) transformed the data sets and results of #1 and #2 to a common geodetic reference ellipsoid as used in the Vincent and Marsh (GSFC) geoid of 1973, and (4) completed the comparative analyses of the resultant geoid profiles against each other and against the Vincent and Marsh geoid. The significant results so obtained have been written up as a formal paper entitled "Geodetic Analysis of Skylab Altimetry Data from SL-2 EREP Pass 9". As an outgrowth and a necessary integral part of our analytical processing and geodetic investigations we expect to prepare a paper discussing Scale and Orientation Control in Geodetic Satellite Altimetry Application and the Effect of Orbit Errors in Satellite Altimetry Geoid Determination.

Documents and data received and reviewed during this period are listed in Appendix A.

## DATA PROCESSING RESULTS

Significant results of data processing and comparative analyses so far completed are contained in Appendix B.

## CONCLUSION

Based on the work completed so far, the technical conclusions are as given in the last progress report and in Appendix B.

As we have previously stated, we feel very strongly encouraged by current data processing results. However, analysis of the results has also identified several implicit problems. Such problems must either be resolved or effectively analyzed in order to (a) arrive at a reliable overall assessment of S-193 altimeter sensor performance evaluation, and (b) indicate the actual contributions toward future satellite altimeter design and programs for earth and ocean physics applications. The achievement of these and similar goals requires the processing and analyses of S-193 altimeter data from all other world sites besides the two test areas involved in our current task.

## PROBLEMS

It appears that due to other priority matters, NASA/Wallops is not in a position to make available to us, in the foreseeable future, their version of SL-2 orbit and altimeter data as we requested through the Technical Monitor. As previously reported and confirmed in Appendix B, NASA/Wallops orbit data appear to be more accurate than that of SKYBET, hence our request for their data. The delays in obtaining additional SL-2 data from NASA/Wallops has put us still further behind our milestone plan. The accuracy problems of SKYBET has led us into investigating "scale and orientation control in geodetic applications of satellite altimetry" and "effect of orbit errors in satellite altimetry geoid determination".

### RECOMMENDATIONS

In view of the above data problems, we have to continue our investigations, based purely on NASA/JSC SKYBET and altimeter data. We, therefore, restate our request for SKYBET tapes as per our letter of November 21, 1973, to the Technical Monitor. The contents of the SKYBET tapes or the "EREP Postpass Summary Reports" are essential for our error analysis, statistical confidence estimates, evaluation of results from repeated EREP pass data, and any necessary rectification of the orientations of the outputs of our investigation.

As a result of all this and the fact that SL-4 data are yet to be received, it is now obvious that our investigations cannot be completed within the original time frame. An extension of time for completion and additional funding will be necessary and are recommended to permit the achievement of the objectives of the investigation.

### NEXT PERIOD AND SUMMARY OUTLOOK

Plans for the next reporting period include

- (1) Continuation of investigation of effect of orbit errors in satellite altimetry geoid determination,
- (2) Continued processing and analysis of the remaining SL-2 data,
- (3) Completion of the investigation and the preparation of a paper on scale and orientation control in geodetic applications of satellite altimetry,
- (4) Preparation for the presentation of an invited paper on "Preliminary Geodetic Processing Results of the Skylab SL-2 Altimeter Data and Potential Applications" at a special EREP session during the annual meeting of the American Geophysical Union in Washington, D. C., April, 1974. This paper is in response to the request of the Technical Monitor and Dick Willmarth of NASA/JSC.

### TRAVEL

No travel was undertaken in this period and none is currently planned for the next period.

APPENDIX AREPORTS AND DATA RECEIVED

	<u>Title</u>	<u>Date</u>	<u>Identification Number</u>	<u>No. of Copies</u>
(1)	Earth Resources Experiment Package (EREP) Experiment Calibration Data		MSC-07744	1
(2)	2 Cans B&W Master "A" Mag. BH01 pas. transp.		Skylab 2 S191 461682 461636	2
	1 each Pos Contact Transparency - BH02 16 MM (Master) Mag: BH02			
(3)	EREP Ground Truth Data for Test Sites (SL-2)	August 15, 1973	NAS8-24000 Amend.JSC-14S	1
(4)	Skylab EREP S191 Infrared Spectrometer Data Acquisition Camera Scene List for SL-3	February 8, 1974	S191 Scene List for SL-3	1
(5)	Quick Update to S193 Rad Scat Raw and Processed Universal Format Data Tapes	February 15, 1974	S193 RS Universal Format Data Tapes	1

In reply refer to: 3K101/Ltr. #74-75

APPENDIX B

GEODETIC ANALYSIS OF SKYLAB ALTIMETRY  
DATA FROM SL-2 EREP PASS 9

(Revision with updated Data of Paper  
Presented at

San Francisco, California

December, 1973)

by

D. M. J. Fubara and A. G. Mourad

February, 1974

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ABSTRACT

GEODETIC ANALYSIS OF SKYLAB ALTIMETRY PRELIMINARY DATA

The analysis was based on a time series intrinsic relationship between the satellite ephemeris, altimeter measured ranges, and the corresponding a priori values of subsatellite geoidal heights. Using, least squares processing with parameter weighting, the objective was to recover (1) the absolute geoidal heights of the subsatellite points, and (2) the associated altimeter calibration constant(s). Preliminary results from Skylab mission SL-2 are given, using various combinations from two sets of orbit ephemeris and altimeter ranges. The influences of orbit accuracy, and a priori geoidal ground truth are described, based on the various combination solutions. It is shown that correctly scaled geoidal heights cannot be deduced by merely subtracting the altimeter range from the geodetic height of the satellite unless the satellite ephemeris and the altimeter have no unknown significant systematic errors or biases and drifts. In particular, the results of such direct subtraction can be very misleading if the orbit used is computed from data that included altimeter data used as height constraints. In view of the current state of our knowledge of (1) satellite altimeter biases and (2) radial errors in orbit computation relative to geocenter, and because satellite altimetry is a "geodetic geometric leveling from space", the use of geodetic ground truth samples as control "benchmarks" appears indispensable for the recovery of absolute geoidal heights with correct scale. Such geodetic ground truth in the oceans have to be determined from marine geodetic techniques involving astrogravimetry and satellite geodesy.

It should be emphasized that the primary objective of the Skylab altimeter is to determine the instrument feasibility. Any additional applications of the data such as for geodesy, geophysics and oceanography are desirable. Although accurate orbit is required for such applications, it is not a pre-requisite for determining the instrument feasibility. The Battelle investigation, nevertheless, considered the influence of orbit accuracy and the effect of other parameters to assess the geodetic requirements for future satellite altimetry missions such as GEOS-C and SEASAT.

ACKNOWLEDGEMENT

The research reported in this paper is sponsored by the National Aeronautics and Space Administration through NASA/Johnson Space Center. The NASA/JSC Technical Monitor is Mr. Z. H. Byrns and the Battelle Principal Investigator is Mr. A. G. Mourad.



GEODETIC ANALYSIS OF SKYLAB ALTIMETRY  
PRELIMINARY DATA - SL/2 EREP PASS 9

INTRODUCTION

The "Williamstown Study" [Kaula, 1970] recommended the use of spacecraft altimeters for geodetic, geophysical and oceanographic studies of the oceans and the earth's gravity field. An effort of this type was implemented for the first time in history under Skylab's experiment S-193, Stanley and McGoogan [1972]. The primary objective of the S-193 is to determine the engineering feasibility of the altimeter. However, Battelle's Columbus Laboratories was awarded a contract for "Calibration and Evaluation of Skylab Altimetry for Geodetic Determination of the Geoid". The S-193 altimeter experiment is one of a number classified under "Earth Resources Experiments Package" (EREP) whose end objectives are to solve various problems on earth, that directly affect even the man in the street.

Three manned Skylab missions--SL/2, SL/3, and SL/4, are to provide data from the S-193 system. Geodetic analysis of Skylab S-193 altimeter preliminary data from mission SL/2 and EREP pass number 9 is the subject of this paper. The overall objective of the Battelle investigation is to demonstrate the feasibility of and necessary conditions in using the altimeter data for determination of the Marine Geoid (i.e., the geoid in ocean areas). The geoid is the equipotential surface that would coincide with "undisturbed" mean sea level of the earth's gravity field. "Undisturbed" is the condition that would exist if the oceans were acted on by the earth's force of gravity only and no other forces such as due to ocean currents, winds, tides, etc. Thus, determination of the geoid (mean sea level) is basic to understanding of the oceans and their dynamic phenomena such as currents, tides, circulation patterns and hence air-sea interactions. Improved numerical weather predictions require accurate knowledge of these ocean dynamics phenomena. Navigation, waste disposal and pollution control also benefit from an accurate knowledge of ocean dynamics. More accurate determination of the geoid will lead to a better definition of the earth's gravity model. Computation of the global geoid by conventional methods is so expensive and time consuming and are beset with so many problems as discussed in Fubara and Mourad [1972a] that these conventional techniques cannot be depended on

for completion of the job in the foreseeable future. These factors justify the need for new systems and techniques. Current indications from the Skylab altimeter are that satellite altimetry may be the answer.

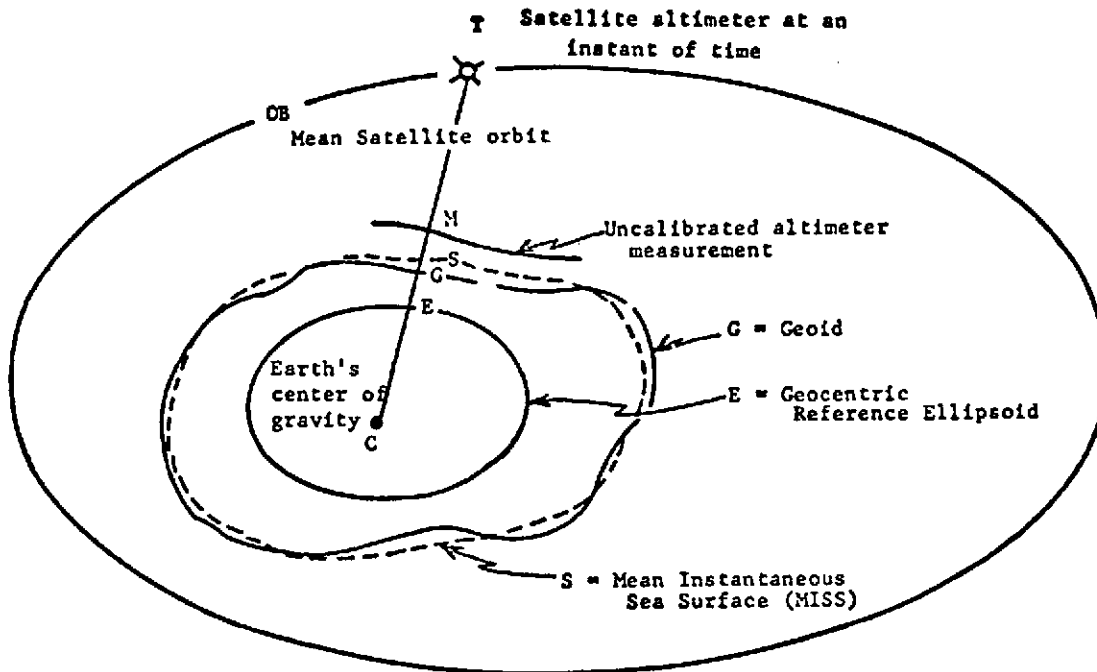


FIGURE 1. SCHEMATIC GEOCENTRIC RELATIONS OF SURFACES INVOLVED IN SATELLITE ALTIMETRY

Figure 1 shows schematic geocentric relations of the various surfaces associated with satellite altimetry.  $TM$  is the raw altimeter range which has to be corrected for laboratory instrumental calibration, electromagnetic effects, sea state, and periodic sea surface influences to give  $TS$ .  $S$  represents the non-periodic "sea level".  $CT$  and  $CE$ , the geocentric radii of the altimeter and  $E$ , its subsatellite point on the reference ellipsoid, are computed from satellite tracking information.  $EG$  is the absolute geoidal undulation to be computed from this investigation, while  $SG$  is the quasi-stationary departure of the mean instantaneous sea surface from the geoid - the "undisturbed" mean sea level.

## ANALYTICAL DATA HANDLING FORMULATIONS

### Condition Equation of Intrinsic Parameters

Each measured altimeter range  $R_i^O$  with an associated measurement residual  $v_i$  is intrinsically related to (1)  $X_s$ ,  $Y_s$  and  $Z_s$  (the satellite coordinates at the instant of measurement), (2) the absolute geoidal undulation  $N_i^a$  (of the subsatellite point) based on a reference ellipsoid of parameters  $a$ , and  $e$ , and (3) the biases in all measurement systems involved. The condition equation for this intrinsic relationship can be stated as:

$$v_i + R_i^O(1 + \Delta c) - h_i + N_i^O + \Delta N_i = 0 \quad (1)$$

where

$\Delta c = f_i$  (systematic errors in  $X_s$ ,  $Y_s$ ,  $Z_s$ , the altimeter bias and sea state correction bias) is the total geodetic calibration constant to be determined. The exact functional mathematical expression for  $\Delta c$  is unknown and is treated later;

$$N_i^a = N_i^O + \Delta N_i \quad (N_i^O \text{ is an approximate value for } N_i^a) = f_2(a, e);$$

and  $h_i$  is the geodetic satellite height above the reference ellipsoid, or

$$h_i = f_2(X_s, Y_s, Z_s, \bar{a}, \bar{e})$$

where  $\bar{a}$  and  $\bar{e}$  are parameters of the reference ellipsoid for the geodetic datum of the tracking stations whose coordinates are used in computing the satellite coordinates  $X_s$ ,  $Y_s$ , and  $Z_s$ . Equation (1) presumes that  $a = \bar{a}$  and  $e = \bar{e}$ ; and also that the two reference ellipsoids are concentric and geocentric.

In current geodetic practice, because of multiplicity of geodetic datums and the non-existence of an universally accepted datum, the  $a = \bar{a}$ , etc. requirements are hardly ever met. A geodetic datum is uniquely determined by seven parameters. One such set of parameters is  $a$ ,  $e$ ,  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ ,  $\Delta \xi$  and  $\Delta \eta$ .

a and e define the size and shape of the reference ellipsoid;  $\Delta x$ ,  $\Delta y$  and  $\Delta z$  relate the center of the reference ellipsoid to the geocenter and are purely translatory; while  $\Delta \xi$  and  $\Delta \eta$  are angular values to ensure parallelity between the minor and major axes of the reference ellipsoid and the mean rotational axis and mean terrestrial equator of the earth respectively. For each geodetic datum, every effort is made to ensure that  $\Delta \xi = \Delta \eta = 0$ . However, as shown in Fubara and Mourad [1972a], this condition has never been exactly realized but its effect can be neglected.

The change  $\Delta h_i$  in  $h_i$  due to the changes  $\Delta a$  and  $\Delta f$  in the dimensions of the reference ellipsoid and  $\Delta x_o$ ,  $\Delta y_o$ , and  $\Delta z_o$  in its position relative to geocenter is given by Heiskanen and Moritz [1967] as

$$\Delta h_i = -\cos \varphi \cos \lambda \Delta x_o - \cos \varphi \sin \lambda \Delta y_o - \sin \varphi \Delta z_o - \Delta a + a \sin^2 \varphi \Delta f \quad (2)$$

where

$$f = \text{flattening of reference ellipsoid } [f = 1 - (1 - e^2)^{1/2}]$$

$\varphi$  and  $\lambda$  = geodetic latitude and longitude corresponding to  $X_s$ ,

$Y_s$ , and  $Z_s$ .

For the current investigation which involves three different geodetic datums, we will further assume that  $\Delta x_o = \Delta y_o = \Delta z_o = 0$  because these values have not been reliably determined and all three datums are supposed to be geocentered. Therefore, instead of Equation (2), we will employ Equation (3)

$$\Delta h_i = -\Delta a + a \sin^2 \varphi \Delta f \quad (3)$$

as the correction parameter to Equation (1) which should be rewritten as

$$v_i + R_i(1 + \Delta c) - (h_i + \Delta h_i) + N_i^0 + \Delta N_i = 0 \quad (4a)$$

or

$$v_i + R_i + \Delta C - (h_i + \Delta h_i) + N_i^0 + \Delta N_i = 0 \quad (4b)$$

to reflect changes in reference ellipsoidal parameters whenever necessary.

$h_i$  is essentially the geodetic height of the satellite above the chosen reference ellipsoid and is given by

$$h_i = (X_s^2 + Y_s^2)^{1/2} \sec \varphi - a(1 - e^2 \sin^2 \varphi)^{-1/2} \quad (5a)$$

or

$$h_i = Z_s \operatorname{Cosec} \varphi - a(1 - e^2 \sin^2 \varphi)^{-1/2} (1 + e^2) \quad (5b)$$

However, usually  $\varphi$  in Equation (5) is not known and has to be derived from

$$\varphi_i = \tan^{-1} \left[ \frac{Z_s + e^2 a (1 - e^2 \sin^2 \varphi)^{-1/2}}{(X_s^2 + Y_s^2)^{1/2}} \right] \quad (6)$$

Equation (6) is usually not solved directly except as recently developed by Paul [1973]. Solving for  $h_i$  and  $\varphi_i$ , from the given  $X_s$ ,  $Y_s$  and  $Z_s$  was done iteratively. By putting  $h_i = 0$ , the first approximation for  $\varphi_i$  is

$$\varphi \approx \tan^{-1} \left[ Z_s (X_s^2 + Y_s^2)^{-1/2} (1 - e^2)^{-1/2} \right] \quad (7)$$

This  $\varphi$  is then used in Equation (6) which is iteratively solved from  $i = 1, \dots, n$  until

$\varphi_n - \varphi_{n-1} \leq \Delta\varphi$  which is usually set at  $\Delta\varphi = 0.001$  arc second. Thereafter,  $h_i$  is computed from Equation (5a) or (5b).

#### Generalized Least Squares Adjustment Model

Equation (1) can be rewritten in matrix form as

$$F_1 (X_1^a, X_2^a, L_1^a) = 0, \quad (8)$$

subject to the normalized weighting functions  $P_1$ ,  $P_2$  and  $P_3$  associated with  $X_1$ ,  $X_2$  and  $L_1$ , respectively. Relating Equations (1) and (8) explicitly,

$$X_1^a = N_i^o + \Delta N_i \quad (9)$$

$$X_2^a = R \Delta c = \Delta C \quad (10)$$

$$L_1^a = R_i^o + V_i \quad (11)$$

In this model, all parameters and measurements of the mathematical model are treated as "measurements" and weighted accordingly. Thus, constants (fixed variables) have infinitely large weights ( $P = \infty$ ) because they need no corrections (residuals) and as residuals tend towards zero, the corresponding weight approaches infinity. Unknown parameters (free variables) in the classical sense have weights  $P = 0$ . All other "measurements" have finite weights  $0 < P < \infty$ . This mathematical model for the generalized least squares processing of experimental data is based on works of Schmid and Schmid [1964], Fubara [1969 and 1973].

The superscript "a" denotes the exact true values of the "measurements". Usually, these true values are not known. Instead, the corresponding measured or approximate values  $X_1^o$ ,  $X_2^o$ , and  $L_1^o$  with associated variance-covariances  $P_1^{-1}$ ,  $P_2^{-1}$ ,  $P_3^{-1}$ , are estimated or measured. Therefore, Equation (8) can be rewritten in the form

$$F_2 \left[ (X_1^o + \Delta_1), (X_2^o + \Delta_2), (L_1^o + V_1) \right] = 0 \quad (12)$$

where

$$X_1^a = X_1^o + \Delta_1$$

$$X_2^a = X_2^o + \Delta_2$$

$$L_1^a = L_1^o + V_1$$

The linearized form of Equation (12) is

$$A_1 \Delta_1 + B_1 \Delta_2 + C_1 V_1 + F_2 (X_1^o, X_2^o, L_1^o) = 0 \quad (13)$$

$A_1$ ,  $B_1$ , and  $C_1$  are the first partial derivatives in a Taylor series expansion of Equation (12), associated with  $X_1^o$ ,  $X_2^o$ , and  $L_1^o$ , respectively, while  $\Delta_1$ ,  $\Delta_2$ , and  $V_1$  are the correction parameters to be determined. The least squares solution of Equation (13) to derive the corrections  $\Delta_1$ ,  $\Delta_2$ , and  $V_1$  to "measured"  $X_1^o$ ,  $X_2^o$  and  $L_1^o$  is as developed in Fubara [1973].

## ANALYSIS AND EVALUATION OF PRELIMINARY DATA

The analytical data handling formulations for this investigation call for the following basic inputs: (1) the altimeter ranges, and exact time (usually GMT) of each measurement to correlate it with (2) the associated orbit ephemeris, and (3) geoidal information used as geodetic control or benchmark along the subsatellite track to help define the geodetic scale of the outputs. The main outputs are: (1) the residual bias of the altimeter or calibration constant required to give a correct absolute geoidal scale, and (2) the geoidal profile, both deduced from the computer processing of the inputs using least squares processing with parameter weighting according to the aforementioned formulations.

Two sets of input data from Skylab mission SL/2, EREP pass #9, are used in this paper. Set A altimeter ranges have been corrected for all known sources of systematic errors including internal calibration constants, refraction and pulsewidth/bandwidth biases. Set B altimeter ranges were not corrected for these specific systematic errors. Figure 2 shows a sample of both sets of ranges. The objectives for processing these two sets are to investigate

- (1) how well the modelling for systematic errors in the analytical data processing procedure can accomodate, recover and prevent such systematic errors from degrading the final results;
- (2) the conditions required to optimally achieve the above objective.

Orbit A data are based on (a) reference ellipsoidal parameters  $a = 6378155$  m. and  $f = 1/298.255$ , (b) SAO 1969 Standard Earth model with geopotential coefficients through degree 22 and order 16, (c) c-band and USB (Unified-S-band) radar tracking data, and (d)  $GM = 3.986013 \times 10^{14} \text{ m}^3/\text{sec}^2$ . Orbit B data are based on (a)  $a = 6378166$  m and  $f = 1/298.3$ , (b) earth gravity model of 3 sectorial and tesseral terms, and 4 zonal terms, (c) C-band and USB radar tracking data, and (d)  $GM = 3.986032 \times 10^{14} \text{ m}^3/\text{sec}^2$ . Both orbits were corrected for other perturbation forces such as lunar gravitation, solar gravitation, earth tide, drag and solar radiation pressure. The geodetic datum for the tracking stations used in each orbit computation was assumed to be geocentric which implies that  $\Delta x_o = \Delta y_o = \Delta z_o = 0$  as described in using Equation (3) instead of (2).

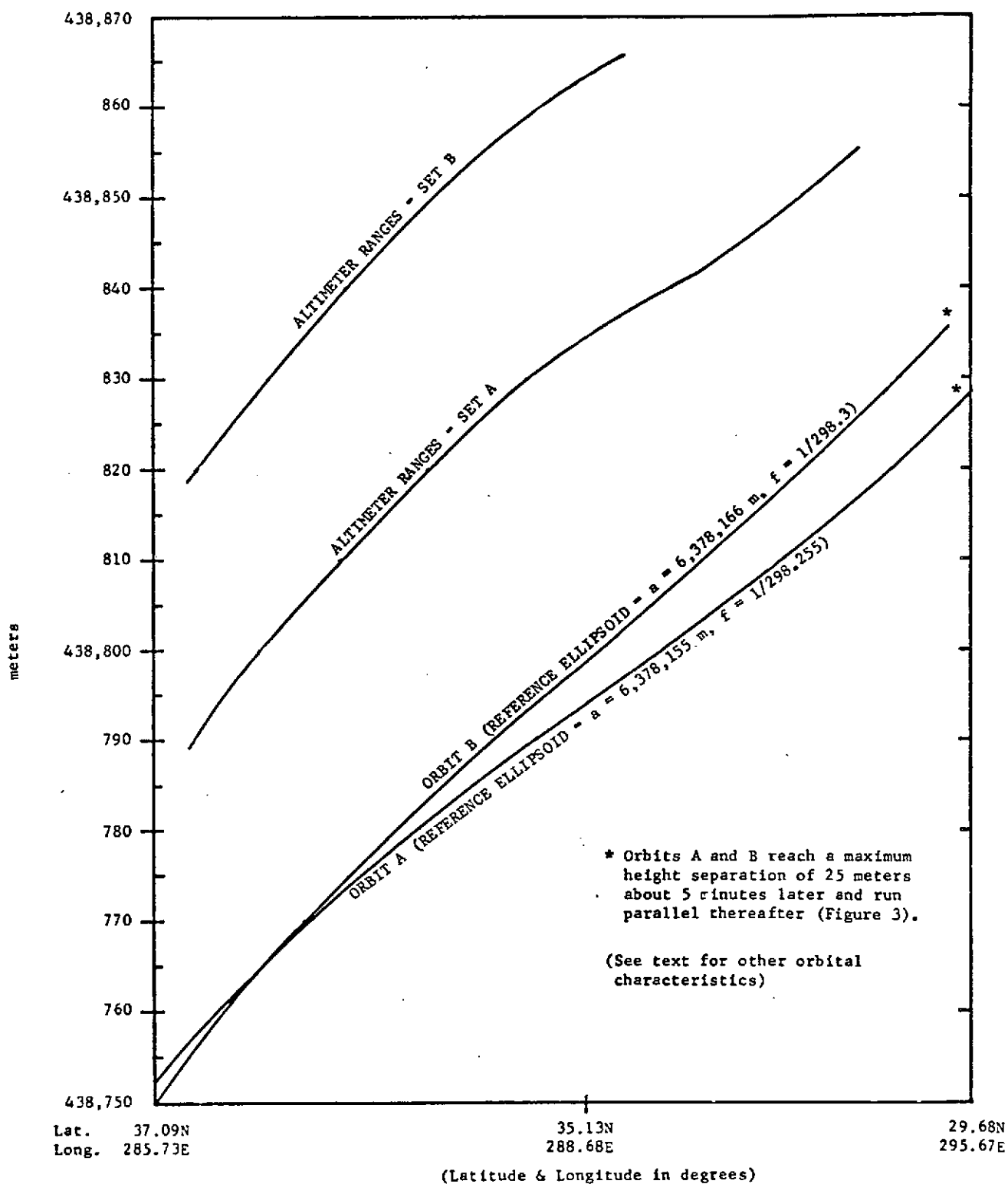


FIGURE 2. ALTIMETER RANGES (MODE 5) AND GEODETIC HEIGHT OF SKYLAB (SL-2 EREP PASS No. 9)  
GMT 13:01:50 to 13:04:50



A segment of each of these two orbits is shown in Figures 2 and 3. In theory, the two orbits should be nearly parallel and radially separated by no more than 11 meters (i.e., the maximum value of  $\Delta h$  of Equation 3). Near to the U.S. east coast, (Figure 2), the two orbits are radially close but not parallel. Further away from the U.S. continent and tracking stations, the orbits diverge to a radial separation of about 25 meters and begin to run parallel (Figure 3). One or a combination of factors including the following may account for these deviations from theoretical expectancy

- (i) one or both of the two geodetic datums of the tracking stations may not be truly geocentric and free of rotational errors as assumed, or there may be undetected systematic errors in individual tracking station geocentric coordinates;
- (ii) the different gravity models influence the computed satellite ephemeris differently. However, the parallelism of the orbit segments away from continental tracking stations is either an accidental coincidence or a reflection that the geometrical constraints of the radar tracking data had ceased to be an influential factor; and
- (iii) differences in orbit computational techniques.

However, it is necessary to point out that by its configuration Skylab is not and was not designed to be a geodetic satellite with highest order tracking systems. Its mass is about 87440 kg. while the "effective" cross-sectional area employed in the orbit computations is  $293.3\text{m}^2$ . In an absolute sense, the computed orbit may not be of geodetic quality. However, it is valid to assume that during short time intervals such as three minutes involved in the data sampling being analyzed, any systematic errors in the orbit will be constant in magnitude and sign. The analytical data processing procedure is designed to effectively accommodate this type of assumption. Therefore, precision wise, the altimeter data and the satellite ephemeris are consistent enough beyond expectations to warrant geodetic analysis.

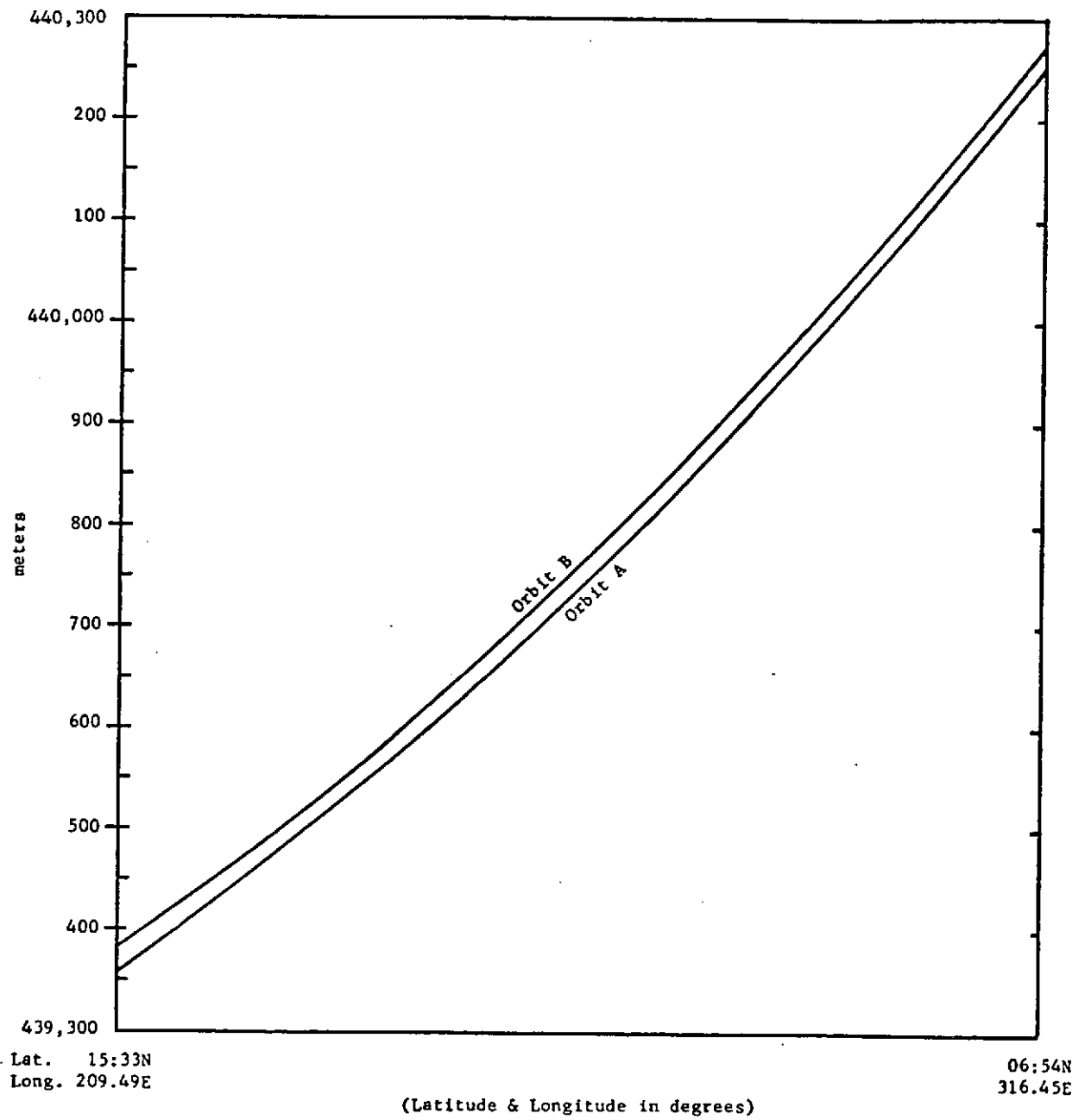


FIGURE 3. GEODETIC HEIGHT OF SKYLAB (SL-2 EREP PASS No. 9)  
GMT 13:10:00 to 13:13:00

The a priori geoid input was taken from Vincent and Marsh [1973] geoid. That geoid is not purely gravimetric as the name implies and therefore, in addition to a flattening of  $f = 1/298.255$ ,  $a = 6378142\text{m}$  is also specified for its reference ellipsoid. To ensure compatibility of geodetic reference datums in Equation (1), Equation (3) was applied as necessary. The two sets of altimeter ranges and orbit ephemeris present four different data combinations that were processed. These various combination solutions were used in the analyses of (1) the efficiency of the data handling formulations, (2) the influences of orbit errors, and (3) the role of the choice of a priori geoidal ground truth. Some schools of thought believe that geoidal heights could be obtained by merely subtracting the altimeter ranges from the corresponding geodetic heights of the satellite. We computed and evaluated results from such a method which we consider invalid because it requires complete absence of systematic errors in the orbit and the altimeter which also must not drift, in order to ensure reliable results.

The Skylab altimeter data being analyzed are from mission SL-2, EREP pass #9 during which data were obtained in Modes 3 and 5 of the instrument's operation. For this pass, there appears to be some instrument malfunction during Mode 3. Therefore, only the Mode 5 data are being analyzed.

### RESULTS AND ANALYSIS

From the given satellite orbit and measured altimeter ranges, the overall objective of the investigation is to simultaneously (a) determine a geodetic calibration constant(s) that (b) corrects or adjusts the altimeter ranges for. (c) determination of absolute geoidal heights with correct scale. Tables 1 to 3 and Figures 2 and 3 show the geodetic heights of the orbits and the altimeter ranges designated as Set A and Set B as previously described. All the results being analyzed have been modified to be based on a reference ellipsoid of  $a = 6,378,142\text{ m.}$  and  $f = 1/298.255$ .

TABLE 1. GEODETIC HEIGHT OF SKYLAB AND A PRIORI  
 GEODAL HEIGHTS INVOLVED IN DATA ANALYSIS  
 (Values are in meters and modified to  
 refer to an ellipsoid  $a = 6378142\text{m}$ ,  
 $f = 1/298.255$ )

Skylab Geodetic Heights Based on		A Priori Geoidal Height Input
Orbit A	Orbit B	
438769.7	438780.5	-41.7
438770.2	438781.2	-41.8
438770.8	438781.8	-42.0
438772.5	438783.4	-42.4
438773.6	438784.8	-42.7
438775.2	438786.7	-43.1
438776.8	438788.6	-43.5
438778.3	438790.4	-43.9
438779.8	438791.8	-44.3
438781.8	438793.7	-44.8
438782.7	438795.3	-45.2
438783.2	438795.9	-45.3
438783.7	438796.4	-45.5
438785.1	438798.1	-45.8
438786.0	438799.0	-46.2
438787.4	438800.6	-46.6
438788.2	438801.6	-46.9
438788.7	438802.1	-47.0
438789.1	438802.7	-47.1
438790.4	438804.5	-47.5
438791.3	438805.5	-47.8
438792.5	438806.7	-48.3
438793.8	438808.2	-48.7
438794.2	438808.9	-48.8
438794.6	438809.4	-49.0
438795.8	438810.6	-49.0
438796.6	438811.7	-49.0
438797.8	438813.0	-49.1
438798.6	438813.8	-49.2
438799.0	438814.3	-49.3
438799.4	438814.8	-49.3
438800.5	438816.3	-49.4
438801.3	438817.2	-49.5
438802.5	438818.4	-49.7
438803.6	438819.9	-49.8
438804.8	438820.3	-50.0
438804.3	438820.7	-49.9
438806.2	438822.8	-49.7
438807.0	438823.8	-49.7
438808.1	438825.3	-49.6
438808.5	438825.6	-49.5
438808.8	438826.0	-49.5
438810.0	438827.4	-49.5

TABLE 2. ANALYTICALLY ADJUSTED RANGES  
 EREP PASS 9 OF SL-2  
 (values in meters)

GMT 13:01:57.981 to 13:02:52.062

Measured Altimeter Ranges		Based on Orbit A Adjusted Altimeter Ranges		Based on Orbit B Adjusted Altimeter Ranges	
SET A	SET B	SET A	SET B	SET A	SET B
438789.1	438818.6	438811.9	438811.9	438824.6	438824.7
438788.7	438819.3	438811.5	438812.6	438824.2	438825.3
438791.0	438819.8	438813.8	438813.2	438826.5	438825.9
438790.6	438821.8	438813.4	438815.2	438826.1	438827.9
438796.2	438823.4	438819.0	438816.7	438831.7	438829.4
438797.0	438825.9	438819.8	438819.3	438832.5	438832.0
438797.7	438827.7	438820.5	438821.0	438833.2	438833.8
438799.6	438829.2	438822.4	438822.5	438835.1	438835.2
438801.1	438831.4	438823.9	438824.8	438836.6	438837.5
438803.3	438832.7	438826.1	438826.0	438838.8	438838.7
438806.3	438835.1	438829.1	438828.5	438841.8	438841.3
438806.3	438835.6	438829.1	438829.0	438841.8	438841.7
438806.3	438836.2	438829.1	438829.6	438841.8	438842.3
438808.2	438837.8	438831.0	438831.1	438843.7	438843.9
438809.3	438838.8	438832.1	438832.2	438844.8	438844.9
438810.8	438840.4	438833.6	438833.8	438846.3	438846.5
438811.2	438840.8	438834.0	438834.2	438846.7	438846.9
438813.1	438841.6	438835.9	438834.9	438848.6	438847.6
438813.5	438842.0	438836.3	438835.4	438849.0	438848.1
438814.2	438844.4	438837.0	438837.7	438849.7	438850.4
438815.7	438845.6	438838.5	438838.9	438851.2	438851.6
438817.2	438846.4	438840.0	438838.8	438852.7	438852.5
438818.7	438848.5	438841.5	438841.8	438854.2	438854.6
438820.2	438849.1	438843.0	438842.5	438855.7	438855.2
438820.6	438849.4	438843.4	438842.8	438856.1	438855.5
Geodetic Calibration Constant					
		22.8	-6.6	35.5	6.1

TABLE 3. ANALYTICALLY ADJUSTED RANGES  
 EREP PASS 9 OF SL-2  
 (values in meters)

GMT 13:02:38.542 to 13:03:33.661

Measured Altimeter Ranges		Based on Orbit A Adjusted Altimeter Ranges		Based on Orbit B Adjusted Altimeter Ranges	
SET A	SET B	SET A	SET B	SET A	SET B
438813.5	438842.0	438836.7	438835.9	438852.3	438851.5
438814.2	438844.4	438837.4	438838.3	438853.0	438853.8
438815.7	438845.6	438838.9	438839.4	438854.5	438855.0
438817.2	438846.4	438840.4	438840.3	438856.0	438855.9
438818.7	438848.5	438841.9	438842.4	438857.5	438857.9
438820.2	438849.1	438843.4	438843.0	438859.0	438858.5
438820.6	438849.4	438843.8	438843.3	438859.4	438858.9
438821.0	438851.3	438844.2	438845.2	438859.8	438860.7
438822.8	438851.8	438846.0	438845.7	438861.6	438861.3
438824.0	438853.2	438847.2	438847.1	438862.8	438862.6
438824.3	438854.3	438847.5	438848.2	438863.1	438863.8
438825.5	438855.1	438848.7	438848.9	438864.3	438864.5
438825.5	438854.6	438848.7	438848.4	438864.3	438864.0
438826.2	438855.7	438849.4	438849.5	438865.0	438865.1
438827.3	438856.8	438850.5	438850.7	438866.1	438866.3
438828.1	438857.9	438851.3	438851.8	438866.9	438867.3
438829.2	438859.7	438852.4	438853.6	438868.0	438869.2
438831.5	438859.9	438854.7	438853.8	438870.3	438869.4
438831.8	438860.3	438855.0	438854.2	438870.6	438869.8
438833.7	438861.9	438856.9	438855.8	438872.5	438871.3
438832.6	438862.7	438855.8	438856.6	438871.4	438872.2
438835.6	438864.4	438858.8	438858.3	438874.4	438873.9
438835.2	438864.6	438858.4	438858.5	438874.0	438874.0
438834.5	438864.1	438857.7	438858.0	438873.3	438873.6
438837.1	438865.7	438860.3	438859.6	438875.9	438875.2
Geodetic Calibration Constant					
		23.2	-6.1	38.8	9.4

### Calibration Constants and Adjusted Altimeter Ranges

As developed earlier, the altimeter bias, radial errors in orbit determination, and errors from inadequate or total lack of correction for significant sea state variations are all algebraically additive. These errors are inseparable unless two of them are absolutely known. In this investigation, the total sum of all three is the geodetic calibration constant to be determined.

Unfortunately, unless the radial orbit error is zero, some known absolute geoidal height must be used as geodetic control or benchmark in order to determine the required geodetic calibration constant. In this case, the calibration constant so determined is scalewise-dependent on the geodetic datum of the a priori geoidal input or the geodetic control used. This is demonstrated in Figure 4. In Figure 4, GG-73 is the subsatellite geoid segment taken from Vincent and Marsh [1973] geoid. AA is the resultant satellite altimetry geoid segment based on GG-73 as a priori input. This a priori input and its output are used as a yardstick or control of the experiment to investigate the effects of errors in a priori geoid height inputs and scale dependency of the computed geodetic calibration constant and satellite altimetry geoid heights on geodetic control (ground truth). Errors were introduced into GG-73 to produce A-I. The resultant satellite altimetry geoid segment from using A-I as a priori input is A-O. Similarly, B-O results from the use of B-I as a priori input.

It is obvious that AA (the control experiment) is shape-wise identical to A-O and B-O. For each case, normalized parameter weighting, consistent with the estimated absolute accuracy of the a priori geoidal height input, was applied. In all cases, even though the resultant point to point geoidal height differences were exactly identical, the deduced calibration constants and hence the values of the computed geoid heights depended on the weighted a priori geoidal height inputs. Figure 4 definitely shows that such a priori inputs and the errors in them affect only the linear scale of the calibration constant and not the shape of the deduced geoid from the type of analytical processing used herein. In other words, the main effect of the a priori geoid input is reflected in the position of the computed geoid relative to geocenter. To determine the geoid with correct shape and scale and centered at geocenter (i.e., an absolute geoid) is the ultimate objective of all geoid computations, and the criteria for the geoid to contribute to solutions of problems in oceanography, geophysics, geodesy and the earth's gravity field model.

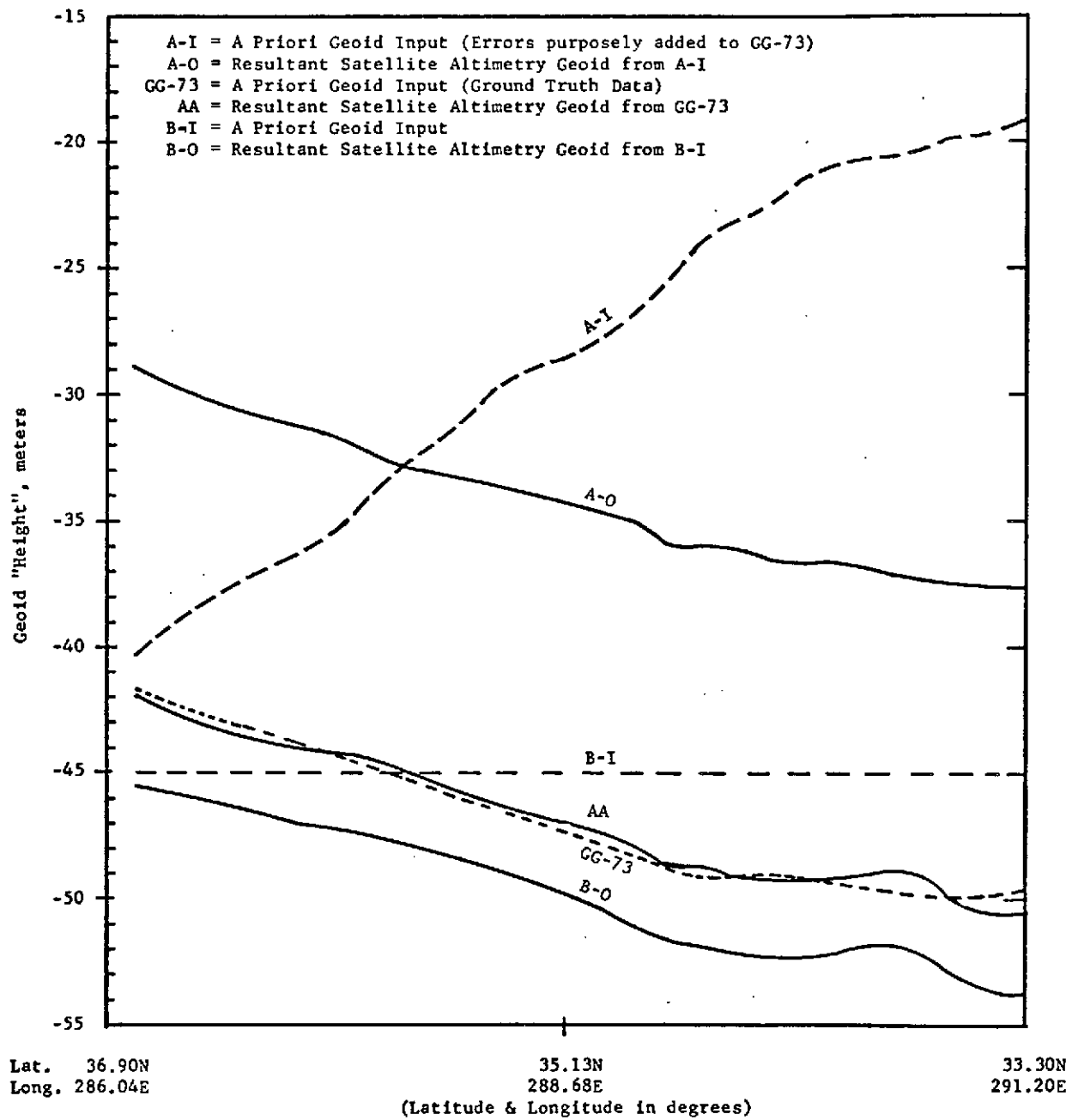


FIGURE 4. EFFECT OF ERRORS IN A PRIORI GEOID HEIGHT INPUTS AND SCALE DEPENDENCY OF CALIBRATION CONSTANT AND GEOIDAL HEIGHT ON GEODETIC CONTROL (GROUND TRUTH)



In the current Skylab data, the altimeter bias appears to vary with the modes and the sub-modes which are described in Kern and Katucki [1973]. This was another factor taken into account. For the current data processing, the additional assumption is that for a "short time interval", the systematic radial orbital errors are of constant magnitude and sign. These two factors constrain the current "short time interval" for this set of data to be no more than 3 minutes. From the calibration constants shown in Tables 2 and 3 the assumption of constant radial orbital errors is better satisfied by Orbit A than Orbit B. For Orbit A, the rate of change in radial errors during this period (close to tracking station) is about 0.5 m per 2 minutes, for Orbit B it is about 3 m per 2 minutes of time. There are currently some avoidable errors in the computation of Orbit B as shown in Wollenhaupt and Schiesser [1973]. In particular the gravity model can be improved. This result supports a well known fact that earth gravity model required for accurate orbit computation is a very important factor.

A key indicator of the reliability of the analytically computed geodetic calibration constant is the consistency of the adjusted ranges. The mathematical model developed for this analysis anticipated imperfections in the knowledge of (1) the orbit and (2) the delay constants (biases) for transforming the radar altimeter returns into ranges in engineering units for geodesy. These problems algebraically add up to be a linear radial error relative to the earth's geocenter. Through the use of the discussed appropriately weighted a priori geoidal heights; (a) no matter what the errors in the different sets of ranges used, the derived adjusted ranges should be identical if the same orbit is used; (b) alternatively, if a unique set of ranges is used with different orbit data, the adjusted set of ranges should differ by only the radial differences between the orbits. The expectations (a) and (b) are established to within the noise level of the data by the results of Tables 2 and 3. Conversely, the deduced geodetic calibration constants should also satisfy condition (b). Thus from Table 2, the constants 22.8 minus 35.5 should equal -6.6 minus 6.1, and from Table 3, 23.2 minus 38.8 should equal -6.1 minus 9.4, meters.

Geoidal Heights Analytically Deduced  
from Satellite Altimetry

Figure 5 shows the deduced geoidal heights from the analytical processing of the four data combinations already described. Figure 5 also shows three other profiles for the same segment of the geoid as given by Vincent, et al [1972 and 1973] using different conventional techniques. As usual, (see Fubara and Mourad [1972] and Fischer, et al [1968]) these other conventional geoid profiles disagree with each other significantly. In Figure 5, GG-72 and GG-73 are conventional geoid segments primarily based on global gravity data which are too sparse and often very inaccurate in ocean areas (70% of the globe) and therefore satellite-derived geopotential coefficients were used to augment the measured gravity data. The present day accuracy and extent of coverage of global gravity data and the geoid are discussed in Decker [1972], and Fubara and Mourad [1973].

By using Orbit A, remarkable agreement achieved (Figure 5) between the analytically computed satellite altimetry geoid segments AA, and AB and GG-73, the Vincent and Marsh [1973] geoid is beyond all expectations. It implies that in the area of the investigation either the GG-73 geoid and Skylab altimeter are extremely accurate or that certain factors have cancelled out to produce such a sub-meter agreement. As has been shown and well accommodated by our analytical data handling model, it is logical to assume that whatever systematic radial errors exist in the computed orbits for the short time period involved, such errors should be constant in magnitude and sign. It is therefore valid to assume that, provided the altimeter system is stable, the deduced altimeter geoid should very closely approximate the true geoid shape of that segment. However, the "absoluteness" scalewise and in orientation of the geoid height is dependent on the orbit and/or the geodetic control that should be used. Such a valid geodetic control or benchmark was not available for this investigation.

The results from using Orbit B shown as segments BB and BA of Figure 5, show a systematic tilt relative to GG-73 and the results based on Orbit A. The main differences between Orbit A and B have been discussed earlier. The conclusion is that the geodetic outcome of satellite altimetry is extremely sensitive to the computed orbit. The agreement between segments AA and AB based on the same orbit but different sets of ranges, one of which set has known systematic errors, shows that our analytical basis is valid and workable for recovery and elimination of the influences of such systematic errors. The same matching applies to BB and BA.

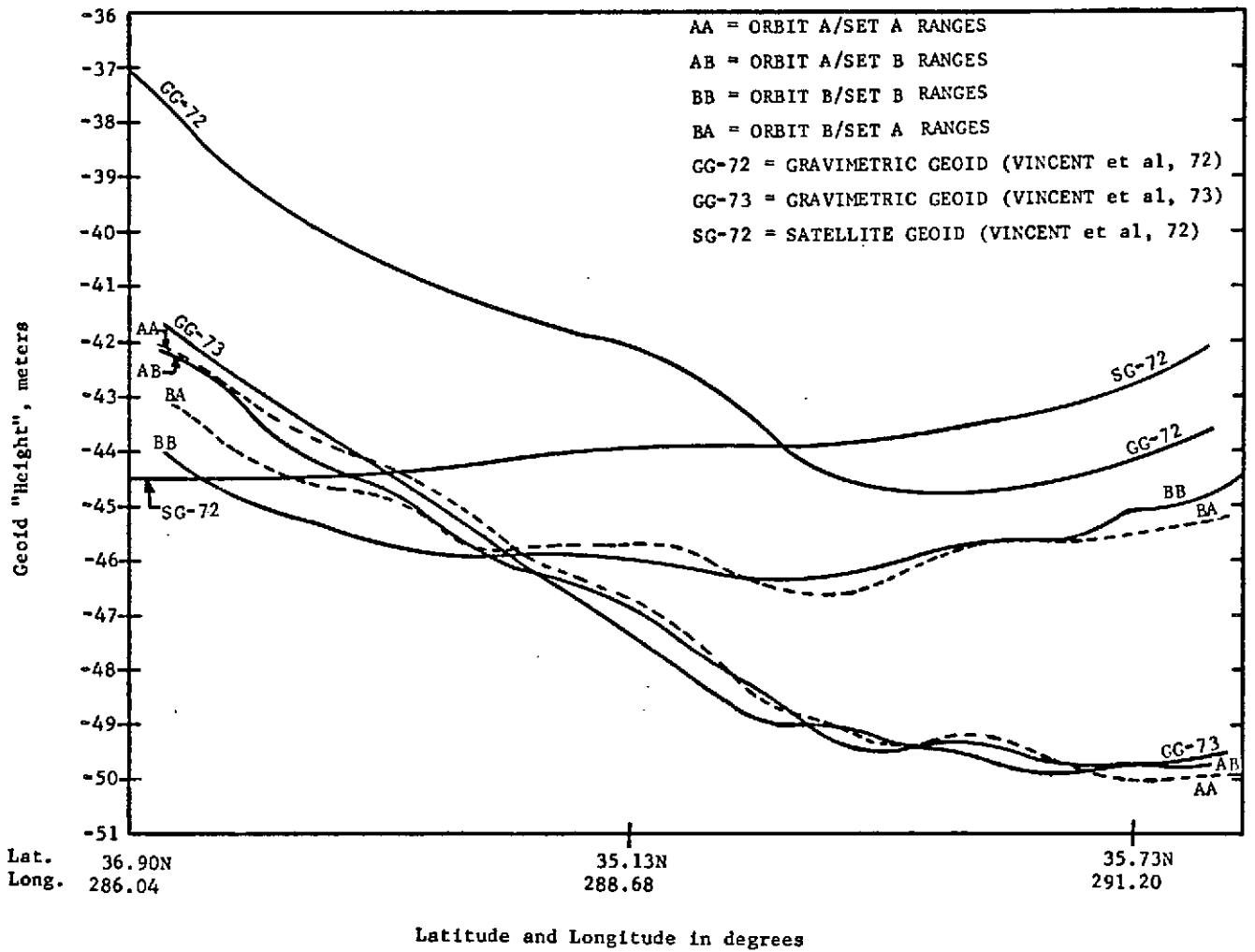


FIGURE 5. CONVENTIONAL GEOID AND SATELLITE ALTIMETRY GEOID SEGEMENTS (SKYLAB SL-2 EREP PASS 9 DATA)

By merely subtracting the measured altimeter ranges from the corresponding satellite geodetic heights, the resultant profiles for the four data combinations are shown in Figure 6. Compared to the results in Figure 5, the simple subtraction results of Figure 6 show, for the Orbit A, remarkable contrast between the "geoid" AA (-19 m to -27.5 m) and AB (-49 m. to -56 m.); for Orbit B and the same two sets of altimeter ranges, "geoid" BB (-38 m. to -40 m. to -39 m.) differ from BA (-8 m. to -11 m. to -10m.). Thus this simple subtraction approach is sensitive not only to the orbit but also to the systematic errors in altimeter ranges unlike the analytical approach. The remarkable match between the analytically computed geoid segments from EREP pass #9, mode 5 data, and Orbit A as given in Figure 5 and the corresponding conventional geoid profile from Vincent and Marsh [1973], as deduced from a combination of terrestrial gravity measurements and satellite-derived geopotential coefficients, should be accepted with caution. Precision estimate of this conventional geoid is about  $\pm 5$  to  $\pm 15$  meters in ocean areas, according to the authors. However, from Rapp [1973], this estimate may be optimistic, in view of certain error sources not accounted for in the computation of that conventional geoid. Furthermore, the segment of the conventional geoid plotted, was scaled off a very small scale world map. This latter process would normally introduce errors into the plotted segment. This condition easily introduces systematic displacement errors which are not conducive to reliable comparison between the two types of geoid segments.

In spite of all these possible sources of discrepancy, and the data errors and uncertainties previously outlined, the comparison of features between the altimetry geoid and this particular conventional geoid (no two conventional geoids are alike and often differ by tens of meters and relative tilts) is very encouraging. The current preliminary results have not been corrected for the influences of sea state, possible nadir alignment errors and departures of the sensor field of view from the nadir. Some of the high frequency features of the satellite altimetry geoid which may be a reflection of these uncorrected influences have been smoothed out. The altimeter ranges refer to some mean sea surface topography of the instant of measurement called MISS in Figure 1. The quasi-stationary departures of the MISS from the geoid is significant in the area of this

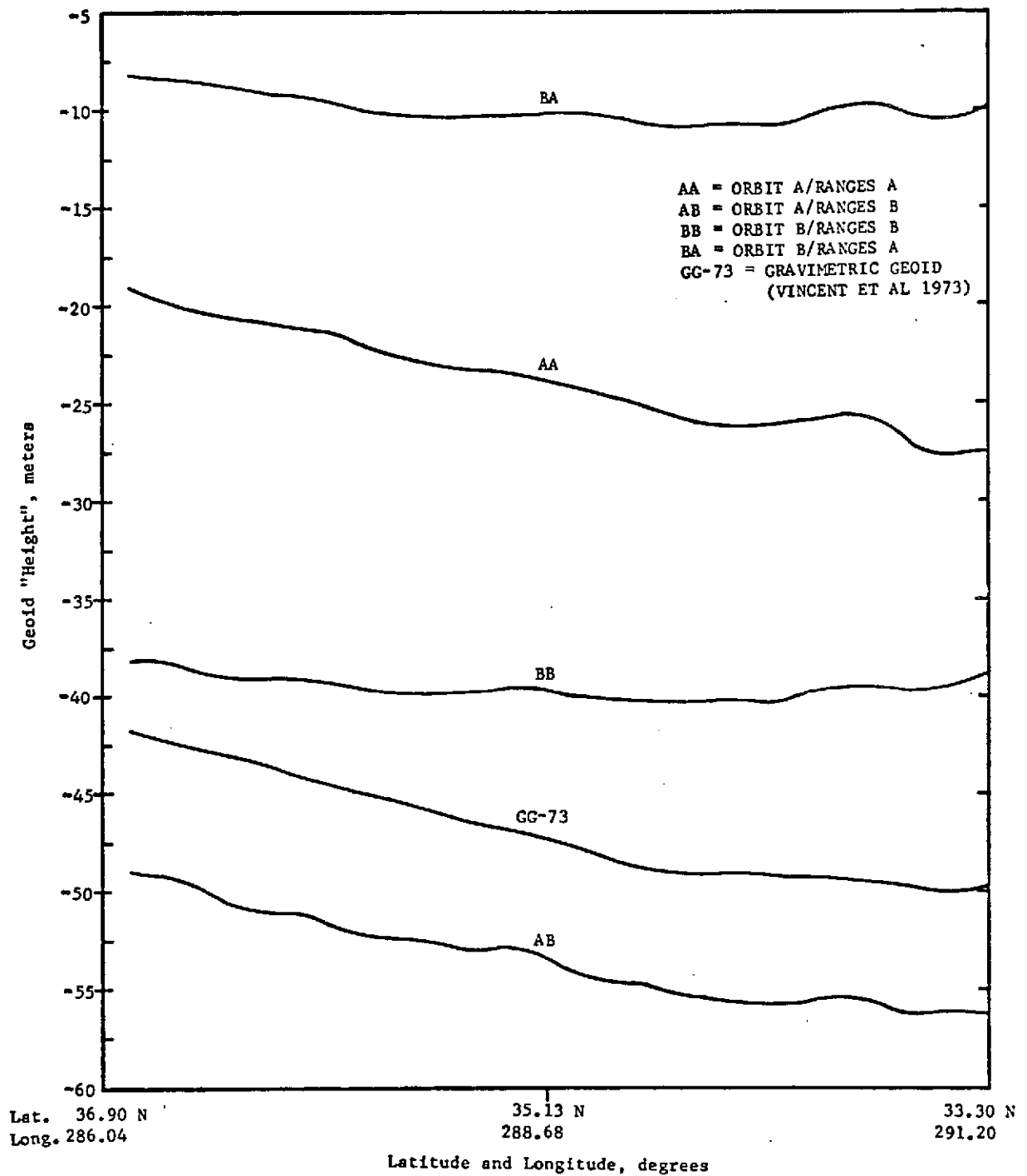


FIGURE 6. SATELLITE HEIGHT MINUS ALTIMETER RANGES AND A CONVENTIONAL GEOID PROFILE (SKYLAB SL-2 EREP PASS 9, MODE 5 DATA)

investigation according to Figures 1 and 2 of Sturges [1972]. If the altimeter is as precise as these results indicate, the expected trend in average sea surface topography of the area could have been sensed. This is being studied further and our computations from EREP pass #4 of mission SL-2 and data expected from mission SL-4 should confirm or negate this expected correlation with sea surface topography.

### CONCLUSIONS

The preliminary conclusions from these quick-look data investigations and previous simulation studies include:

- (1) The analytical data handling formulations developed for this investigation appear to be very satisfactory. The main outputs required, the geodetic calibration constant, the geoid height and the corrected altimeter ranges were reliably determined;
- (2) To ensure that the deduced calibration constant and geodetic heights are absolute, the use of geodetic control or a benchmark whose absolute geoidal height is known is indispensable. The establishment of such controls from a combination of astrogravimetry and satellite data is discussed in Mourad and Fubara [1972], and in Fubara and Mourad [1972a] and the practical implementation is partially demonstrated in Fubara and Mourad [1972b]. There is an implicit correlation between this conclusion and the conclusion based on a different type of investigation in Rapp [1971] that: "In carrying out simulation studies with non-global data it was concluded that altimetry data could not be used alone for potential coefficient determination.... Consequently, the altimetry data was combined with geoid undulation information in non-ocean blocks and with existing terrestrial gravity data.";

- (3) On the assumption that the altimeter system is stable, and that systematic orbit radial errors for short time periods are constant, the altimeter geoid shows very high frequency details which have been smoothed out in the plotten geoid or more accurately the sea surface topography. Such high frequency details may also reflect the inexact fulfillment of various assumptions implied or the uncorrected influence of sea state.
- (4) Subject to additional data processing corrections which the current state of the SL/2 data precludes, these preliminary results indicate that satellite altimetry will be a valid and useful tool for computing quasi-stationary departures of sea surface topography from the geoid. This practical application is important to oceanographic work related to ocean dynamic phenomena such as circulation patterns, mass water transport, ocean tides, ocean current influences, etc. These in turn relate to air-sea interaction and the knowledge for global numerical weather prediction. Such oceanographic factors also affect our knowledge of pollution dispersion by the oceans, an important guiding factor in waste disposal, and prediction of dispersal and control of oil spill hazards. Further developments on these issues are in Fubara and Mourad [1973];
- (5) Orbit computation in which inadequately calibrated altimeter ranges are employed as constraints is not desirable and present no advantage for processing altimeter data to compute the geoid. First, the unmodelled range biases introduce large systematic errors that are not admissible in least squares orbit computation. Such systematic errors cannot be accurately eliminated through modeling unless some valid geodetic controls are used as constraints. Second, the use of orbits computed in this way to deduce a geoid from the same altimeter data with purely differencing or graphical techniques would be misleading. For example, the geoid so deduced would closely match the original geoid used in applying the altimeter ranges as a constraint in the orbit computation.

- (6) Deduction of a correctly scaled geoid from satellite altimetry cannot be achieved by merely subtracting altimeter ranges from the corresponding geodetic heights of the satellite unless (a) the satellite orbit is errorless, (b) the altimeter does not drift, and (c) the altimeter system biases are either non-existent or are absolutely known. Therefore, in practice, at this time, satellite altimetry ranges cannot be regarded as representing direct determination of absolute geoid heights as one would like to assume. At this time marine geodesy, involving the use of astrogravimetric and satellite geodesy techniques, appears indispensable for the provision of geodetic controls required for the full achievement of satellite altimetry objectives of GEOS-C, and SEASAT series of the NASA-proposed "Earth and Ocean Physics Applications Program".



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